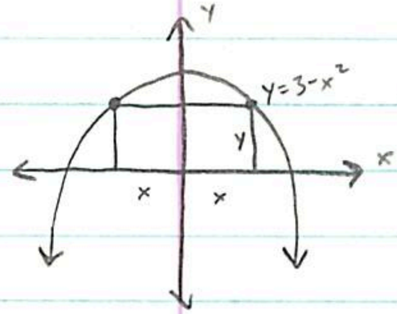
Calc Q3T1 Review Key

1)



Maximize Area

$A = 2xy$

$A = 2x(3-x^2)$

$A = 6x - 2x^3$

$A' = 6 - 6x^2$

Restrictions

$x > 0$

$y > 0 \qquad 3 - x^2 > 0$

$3 > x^2$

$\sqrt{3} > x$

$0 < x < \sqrt{3}$

$A' =$ undefined
Never

$A' = 0$

$0 = 6 - 6x^2$

$0 = 6(1-x^2)$

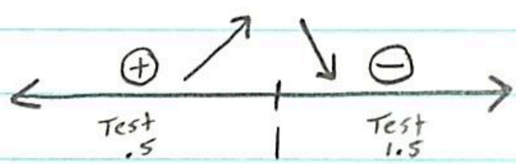
$0 = 6(1+x)(1-x)$

$x = -1$ reject $\quad x = 1$

$A'(.5) = 6 - 6(.5)^2$
$= 4.5$ (Positive)

$A'(1.5) = 6 - 6(1.5)^2$
$= -7.5$ negative

A'

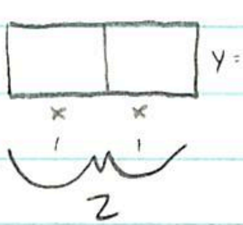


Maximum @ $x = 1$

Dimensions: recall $y = 3 - x^2$

$y = 3 - 1^2$

$y = 2$



$\boxed{2 \text{ by } 2}$ so Max Area $= \boxed{4}$

2) Let $x = $ 1st #

$y = $ 2nd #

Minimize Sum of squares

$$S = x^2 + y^2$$

$$x + y = 96$$

$$y = 96 - x$$

$$S = x^2 + (96-x)^2$$

$$S = x^2 + 9216 - 192x + x^2$$

$$S = 2x^2 - 192x + 9216$$

$$S' = 4x - 192$$

$S' =$ undefined

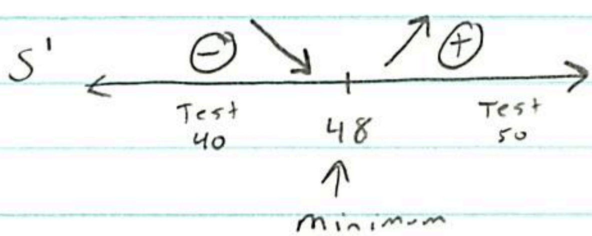
Never

$$S' = 0$$

$$0 = 4x - 192$$

$$192 = 4x$$

$$48 = x$$



$$S'(40) = 4(40) - 192$$

$$= -32 \text{ (negative)}$$

$$S'(50) = 4(50) - 192$$

$$= 8 \text{ (positive)}$$

$$x = 48$$

$$y = 96 - x = 96 - 48 = 48$$

$$\boxed{x = 48 \qquad y = 48}$$

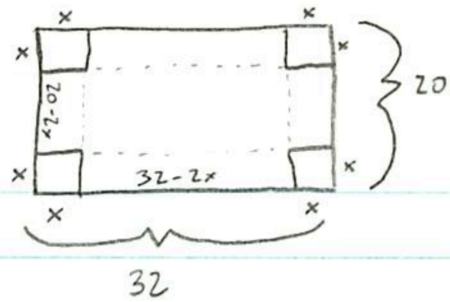
Minimum Sum

$$= 48^2 + 48^2$$

$$= 2304 + 2304$$

$$= \boxed{4608}$$

3)



Restrictions

$x > 0$

$20 - 2x > 0$

$20 > 2x$

$10 > x$

$0 < x < 10$

Maximize Volume

$V = L \times W \times H$

$V = (32-2x)(20-2x)(x)$

$V = (640 - 64x - 40x + 4x^2)(x)$

$V = 640x - 104x^2 + 4x^3$

$V' = 640 - 208x + 12x^2$

$V' =$ undefined

Never

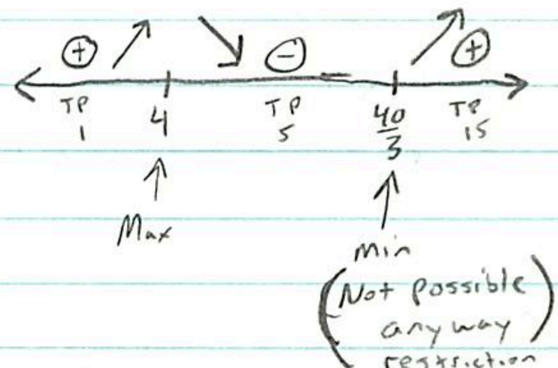
$V' = 0$

$0 = 12x^2 - 208x + 640$ use Quad Formula

$x = \frac{40}{3}$ or $x = 4$

13.333

V'



$V'(1) = 640 - 208(1) + 12(1)^2$

$= 444$ (pos)

$V'(5) = 640 - 208(5) + 12(5)^2$

$= -100$ (neg)

$V'(15) = 640 - 208(15) + 12(15)^2$

$= 220$ (pos

Length of squares

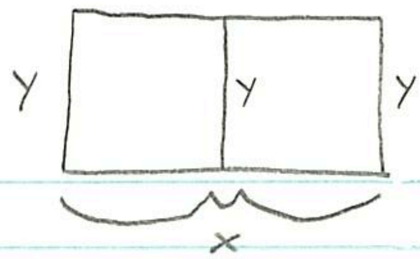
$x = 4$ inches

Max Volume

$V = 640(4) - 104(4)^2 + 4(4)^3$

$= \boxed{1152 \text{ inches}^3}$

4)



Minimize Perimeter

$P = 2x + 3y$

$P = 2x + 3\left(\frac{6144}{x}\right)$

$P = 2x + \frac{18432}{x}$

$\underline{\text{Area}} = xy$

$6144 = xy$

$\frac{6144}{x} = y$

restrictions

$x > 0$

$y > 0$ so $\frac{6144}{x} > 0$

$P = 2x + 18432x^{-1}$

$P' = 2 - 18432x^{-2}$

$P' = 2 - \frac{18432}{x^2}$

$P' = \text{undefined}$

$x = 0$

reject (restriction)

$P' = 0$

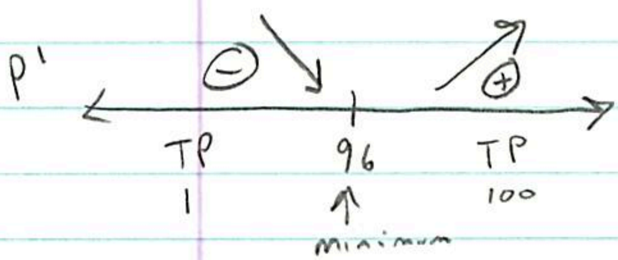
$0 = 2 - \frac{18432}{x^2}$

$0 = 2x^2 - 18432$

$18432 = 2x^2$

$9216 = x^2$

$96 = x$



$x = 96$

$y = \frac{6144}{96} = 64$

$P'(1) = 2 - \frac{18432}{1^2}$

$= 2 - 18432$

$= -18430$ (neg)

$P'(100) = 2 - \frac{18432}{100^2}$

$= 2 - \frac{18432}{10000}$

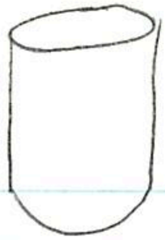
$= 2 - 1.8432$

$= .1568$ (pos)

Perimeter $= 2x + 3y$

$= 2(96) + 3(64)$

$= \boxed{384}$ feet of fence

5) 

$V = \pi r^2 h$

$64\pi = \pi r^2 h$

$64 = r^2 h$

$\frac{64}{r^2} = h$

Minimize Surface Area

$SA = 2\pi r^2 + 2\pi r h$

$SA = 2\pi r^2 + 2\pi r\left(\frac{64}{r^2}\right)$

$SA = 2\pi r^2 + \frac{128\pi}{r}$

$SA = 2\pi r^2 + 128\pi r^{-1}$

$SA' = 4\pi r - 128\pi r^{-2}$

$SA' = 4\pi r - \frac{128\pi}{r^2}$

$SA' =$ undefined

$r = 0$ reject (need a radius)

$SA' = 0$

$0 = 4\pi r - \frac{128\pi}{r^2}$

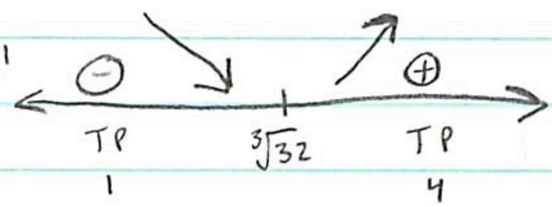
$\frac{128\pi}{r^2} = 4\pi r$

$\frac{128\pi}{4\pi} = \frac{4\pi r^3}{4\pi}$

$32 = r^3$

$\sqrt[3]{32} = r$

$3.17 \approx r$



$SA'(1) = 4\pi(1) - \frac{128\pi}{1^2} = 4\pi - 128\pi = -124\pi$ (neg)

$SA'(4) = 4\pi(4) - \frac{128\pi}{4^2} = 16\pi - 8\pi = 8\pi$ (pos)

Minimum SA when $r = \sqrt[3]{32} \approx 3.17$

$h = \frac{64}{(\sqrt[3]{32})^2} \approx 6.35$

Min SA

$= 2\pi\left(\sqrt[3]{32}\right)^2 + 2\pi\left(\sqrt[3]{32}\right)\left(\frac{64}{\sqrt[3]{32}^2}\right)$

$= \boxed{189.99 \text{ in}^2}$

6) $C(x) = x^2 - 40x + 520$

$C' = 2x - 40$

$c' =$ undefined

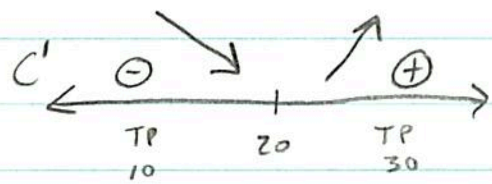
Never

$c' = 0$

$0 = 2x - 40$

$0 = 2(x - 20)$

$x = 20$



$C'(10) = 2(10) - 40 = -20$

$C'(30) = 2(30) - 40 = 20$

Speed = $\boxed{20}$

$C(20) = 20^2 - 40(20) + 520$

Cost = $\boxed{\$120}$